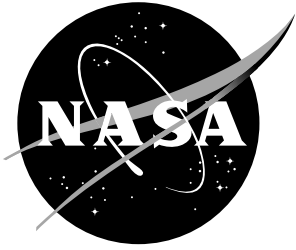# On the Importance of Cycle Minimum in Sunspot Cycle Prediction

Robert M. Wilson, David H. Hathaway, and Edwin J. Reichmann

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ON THE IMPORTANCE OF CYCLE MINIMUM IN SUNSPOT CYCLE PREDICTION

I.   INTRODUCTION

The current sunspot cycle, cycle 22, began in 1986, peaked in 1989, and is now in its 10th year. Based on annual averages for cycles 0 to 22, cycle 22 has the highest minimum amplitude ($Rmin$ = 13.4) and the second highest maximum amplitude ($Rmax$ = 157.6) on record, and its rise to maximum (3 years) is among the fastest. Similarly, in terms of smoothed sunspot number (i.e., 12-month moving averages), cycle 22 began in September 1986, peaked in July 1989, and, with the start of April 1996, entered its 115th month. Again, comparison to other sunspot cycles reveals that it has the highest minimum amplitude (Rm = 12.3), the third highest maximum amplitude (RM = 158.5; tied with cycle 3), and is the fastest rising cycle (ASC = 34 months) on record.

Recent studies by Wilson[1] [2] and Wilson et al.[3] strongly suggest that cycle 22 is a short-period cycle (Rabin et al.[4] and Wilson[5] [6]), with onset for cycle 23 expected before April 1997, probably near December 1996 ($\pm$3 months). Furthermore, Hathaway et al.[7] have shown that the size and placement of cycle minimum is crucial for the accurate forecast of solar activity, especially with regard to the size and placement of cycle maximum and to the declining phase of the cycle. In this paper, several features of the sunspot cycle that relate to cycle minimum and to the prediction of sunspot maximum are examined more closely, in particular, with application to cycle 23.

II.   RESULTS

Table 1 summarizes the various statistical aspects of the sunspot cycle for cycles 0 to 22 (adapted from data published in Waldmeier[8] and McKinnon,[9] and updated from monthly mean sunspot number values published in "Solar Geophysical Data," available from the National Oceanic and Atmospheric Administration, National Geophysical Data Center, Boulder, CO). Tabulated as a function of sunspot cycle number are the epochs of sunspot minimum and maximum and their amplitudes, based on both smoothed sunspot numbers and annual averages, and their ascent and descent durations and periods in months for the smoothed data and in years for the annually averaged data. From table 1 (ignoring the division of the sunspot record into a pre- and post-modern sunspot era, based on the completeness of sunspot data), it is found that smoothed sunspot minimum amplitude has ranged from 0.0 to 12.3, smoothed sunspot maximum amplitude from 48.7 to 201.3, ascent duration from 34 to 81 months, descent duration from 48 to 123 months, and period from 108 to 164 months. Similarly, it is found that annual average minimum amplitude has ranged from 0.0 to 13.4, annual average maximum amplitude from 45.8 to 190.2, ascent duration from 3 to 7 years, descent duration from 5 to 11 years, and period from 9 to 14 years. Values for cycle 22 (the present cycle) remain incomplete, owing to the lack of a discernible onset for cycle 23.

From table 1, it is noted that fast-rising cycles (ascent duration <4 years) tend to be better associated with cycles of larger than average RM, while slow-rising cycles tend to be better associated with cycles of smaller than average RM. This relationship between the ascent duration of the cycle and its maximum amplitude is often known as the Waldmeier effect (Bracewell[10] and Wilson[11]) and is shown in figure 1 (left panel), both in terms of smoothed (top) and annually averaged (bottom) data. The probability of obtaining the observed distributions, or ones more suggestive of a departure from independence (chance), is P < 1 percent. Therefore, given that a cycle is known to be of larger (smaller) than average maximum amplitude, one can infer that it will also be fast (slow) rising, or vice versa. Such a relationship has been true for 20 of 22 sunspot cycles.

The relationship between maximum amplitude and ascent duration can also be described using the regression fit given in figure 1 (left-top panel), which shows that about 54 percent of the variance can be explained by the fit. Based on the regression fit, it is found (right panel) that, given the observed value of maximum amplitude for a cycle, the observed value for the ascent duration usually lies within 15 percent of its predicted value, true for 14 of 22 sunspot cycles; 20 of 22 cycles are noted to have their observed values within 30 percent of their predicted values. As an example, if one suspects that a cycle will have an *RM* of about 160, from the regression, it is inferred that its *ASC* should be about 39±6 months for the ±15-percent range or about 39±12 months for the more inclusive ±30-percent range.

Close inspection of table 1 (annual averages) also reveals another interesting aspect of fast-rising cycles. Namely, not only do fast risers (*asc* < 4 years) tend to be better associated with larger than average maximum amplitude (8 of 8), but they more often have been found to be better associated with cycles of shorter than average period (5 of 7; the period of cycle 22 is not yet known). Of the 7 examples of cycles with *asc* = 3 years, two had *per* = 9 years (cycles 2 and 3), three had *per* = 10 years (cycles 17, 18, and 21), one had *per* = 11 years (cycle 11), and one had *per* = 14 years (cycle 4). Because cycle 22 also has an *asc* = 3 years, it is strongly suspected that it, likewise, will have *per* < 11 years. Recent work by Wilson,[1] [2] in fact, strongly supports the notion that cycle 22, indeed, will be of shorter than average length; hence, an *Emin* of 1996 for cycle 23 seems very likely.

While a meaningful (i.e., statistically significant) regression is not apparent between *RM* and *PER* for the same cycle *n* (in contrast to that reported by Schatten et al.[12]), one is found when *RM* (cycle *n*) is compared against *PER* (cycle *n*–1). This amplitude-period effect, which was previously noted by Hathaway et al.,[7] is depicted in figure 2. Based on the observed 2 by 2 contingency table, it is found that the probability of obtaining the observed result, or one more suggestive of a departure from independence, is *P* = 2.3 percent. Thus, when the immediately preceding cycle is of short duration (*PER* < 132 months), *RM* for the current cycle is usually larger than average, while, when the length of the preceding cycle is of long duration, *RM* for the current cycle is usually smaller than average. Such a relationship has been true for 16 of 21 sunspot cycles. For the three regression fits (all; all excluding 16, 19, and 21; and modern only) shown in figure 2 (right panel), it is found that the observed value of *RM* typically lies within 30 percent of its predicted *RM*. As an example, if one measures a cycle's length to be about 120 months, one would expect the following cycle to have an *RM* greater than average in size and to be about *RM* = 140±42 (for the ±30-percent range).

Figure 3 displays another seemingly important relationship involving minimum amplitude; namely, the maximum-minimum effect. Given that a cycle has a larger (smaller) than average size minimum amplitude, one can infer that it usually will also have a larger (smaller) than average size maximum amplitude. This has been true for 16 of the 22 sunspot cycles. The probability of obtaining the observed distribution, or one more suggestive of a departure from independence, is *P* = 4.3 percent. A comparison of observed and predicted *RM* values, based on the regression fits shown in figure 3 (right panel), shows that the later-occurring *RM* typically lies within 30 percent of its predicted value. Again, as an example, if a cycle has an observed minimum amplitude *Rm* = 10, one would expect the later-occurring maximum amplitude *RM* to measure about 140±42 (for the ±30-percent range).

III.   DISCUSSION AND CONCLUSIONS

Figure 4 depicts cycle 22 from onset (1986) through its ninth year (1995) of elapsed time from *Emin* (0) and shows the relative placement of the epochs of conventional smoothed sunspot number minimum (*Em*) and maximum (*EM*) with respect to *Emin* and *Emax* years. Based upon the values of sunspot number now being reported at the start of 1996 and the observed range of observed *Rmin* values for cycles 1 to 22 (0.0 to 13.4), it appears that *Emin* for cycle 23 will be 1996. Support for this also comes from the inferred average slope in sunspot number during the decline of cycle 22 (e.g., Wilson[11] and Wilson et al.[3]) and from the first occurrence of a spotless day during the decline of cycle 22 (Wilson[2]).

Thus, presuming that *Emin* for cycle 23 will be 1996, one infers that *Em* should occur sometime between December 1995 and March 1997, with the most likely date being December 1996±3 months (Wilson et al.[3]).

Furthermore, presuming that *Em* for cycle 23 occurs before September 1997, it follows that cycle 22 is a short-period cycle (*PER* < 132 months), and, from the amplitude-period effect, one can infer that the maximum amplitude for cycle 23 will probably be larger than average in size. Likewise, presuming that the maximum amplitude for cycle 23 will be larger than average in size, from the Waldmeier effect, one can infer that it probably will be a fast-rising cycle. Hence, *Emax* should follow *Emin* by 3 years (or, less likely, 4 years—8 of 12 large amplitude cycles have had *asc* = 3 years and 3 of 12 have had *asc* = 4 years; see fig. 1), suggesting that *Emax* for cycle 23 may very well occur in 1999. Consequently, from figure 4, one infers that *EM* for cycle 23 should occur sometime between November 1998 and March 2000. If, on the other hand, *Emax* for cycle 23 turns out to be the year 2000, then *EM* would be expected to occur between November 1999 and March 2001.

Additional corroborating evidence, supporting the claim that cycle 23 may be a large-amplitude cycle, will be available once *Rm* has finally been measured (from the maximum-minimum effect). Note, however, that *Rm* for cycle 23 probably will be large, and that, consequently, its *RM* should be larger than average, as well. This is gleaned, not only from the current behavior of the decline of cycle 22 (from figure 4 and Wilson et al.[3]) and the aforementioned effects, but also from the behavior of the first difference of *Rm* and *RM*, shown in figure 5.

The first difference (*fd*) of a parameter is computed as parametric value for cycle n+1 minus parametric value for cycle *n*. The first differences for *Rm* and *RM* (top and bottom panels of fig. 5, respectively) display behaviors that appear to be statistically important (supportive of an even-odd cycle effect in the sunspot record). For each parameter, the central 50-percent interval is identified, inferring that about half of the sunspot cycles have first differences that fall within the central 50-percent spread. Thus, for *Rm*, its central 50-percent spread is inferred to be about ±3 units, and for *RM*, its central 50-percent spread is inferred to be about ±40 units. It follows then that, because the *Rm* and *RM* values for cycle 22 are already known, one can easily calculate the expected 50-percent probability range of *Rm* and *RM* for cycle 23; namely, $Rm = 12.3\pm3$ and $RM = 158.5\pm40$. Thus, for cycle 23, there is about a 50:50 chance that its *Rm* will be between 9 and 15, or about a 75-percent chance that it will be either above ($Rm > 9.3$) or below ($Rm < 15.3$) the extremes of the prediction limits. For *RM*, there is about a 50:50 chance that its value will be between about 120 and 200, or about a 75-percent chance that it will be either above ($RM > 118.5$) or below ($RM < 198.5$) the extremes of the prediction limits. A more restrictive range for *Rm* and *RM* for cycle 23 is found, however, when *fd* is separated into even- and odd-numbered cycle groupings. For the modern era, the average *fd* for an even-numbered cycle is 0.0 units for *Rm* (having a standard deviation of 2.9 units) and 40.3 units for *RM* (having a standard deviation of 14.2 units). Therefore, minimum and maximum amplitudes for cycle 23 can be easily computed as $Rm(23) = Rm(22)+0.0\pm7.5$ and $RM(23) = RM(22)+40.3\pm36.5$, where $Rm(22) = 12.3$, $RM(22) = 158.5$, and the prediction intervals refer to the 95-percent level of confidence limits. Thus, $Rm(23) = 12.3\pm7.5$ and $RM(23) = 198.8\pm36.5$. Looking at the lower extremes, one finds that $Rm(23) > 4.8$ and $RM > 162.3$ at the 97.5-percent level of confidence, and, for the upper extremes one finds $Rm(23) < 19.8$ and $RM(23) < 235.3$ (Kopecký [13] and Wilson[14]). Hence, based on the expected *fd* values for cycle 22 (using only the modern era cycles), we infer that maximum amplitude for cycle 23 will possibly be near record value, either the second largest or, perhaps, the largest cycle ever observed!

The above findings may be somewhat surprising in that, recently Schatten et al.[15] have forecast cycle 23 to have a 1-sigma prediction interval for *RM* equal to about 138±30 and an *EM* of about May 2000 (±9 months), somewhat different from what had been predicted for it earlier (Schatten and Pesnell[12]) based on the same prediction technique (the so-called Solar Dynamo Amplitude or SODA index); namely, a size comparable to that of cycle 22 and an *EM* of about 1999.7±1 year. Some comments seem to be in order. First, concerning their prediction of the size of cycle 23, it should be noted that it too predicts that cycle 23 will be larger than average. Presuming that ±30, indeed, is the 1-sigma standard error (based upon the most recent 10 sunspot cycles), one finds that the prediction interval at the 95-percent level of

confidence based on the SODA index should be 138±69.2, where the prediction interval limits are 2.306 times 30 (assuming 8 degrees of freedom, the *t* value is 2.306). Presuming that the bulk of the cycles have their observed *RM* within 30 percent of their predicted value (as found in this study using the amplitude-period effect or the maximum-minimum effect), it is inferred that their prediction should be about 138±41 units. This value, in fact, is very close to that which is found based on the amplitude-period effect and maximum-minimum effect, using a period of about 120 months for cycle 22 and an expected *Rm* of about 10 for cycle 23.

Second, concerning their prediction of *EM* for cycle 23, because they too predict that cycle 23 will be above average in size, based on the Waldmeier effect (true for 20 of 22 cycles), one should expect *EM* to follow *Em* by <48 months and *Emax* to follow *Emin* by <4 years. In their study, however, Schatten et al.[15] ignore the occurrence of sunspot minimum. Instead, they estimate the timing of sunspot maximum based on an inferred relationship of the time differences between occurrences of solar maxima and the last 16° latitude appearance versus maximum sunspot number, and on the presumption that the period of a cycle is linearly related to the amplitude of the cycle at that instance. Because the last 16° latitude crossing was January 1991 and the sunspot amplitude at that instance measured 147.6, they deduce that the time to the next last crossing of the 16° latitude occurrence will be about 123 months later (a value that happens to be the average length of a short-period cycle, based on a bimodal distribution of cycle lengths; see Wilson[5] and Wilson et al.[3]). They then deduce that the offset to this period should be about –11 months, based on their prediction of the size of cycle 23 (RM = 138). Hence, they deduce that *EM* should occur about January 1991+123 months–11 months, or that it should occur about May 2000 (with a stated uncertainty of about 9 months). (A word of caution, however, seems warranted in that the stated standard error for their offset is 5.7 months, implying that the 95-percent level of confidence limits on their prediction, employing their offset, for a sample size of 11 cycles is about 13 months! So, *EM* could come as much as 1 year either earlier or later than the predicted date for *EM*.)

In some respects, this date seems somewhat later than it should be, while in others it seems very close to that which one should expect. For example, presuming cycle 23 to have *Emin* in 1996 and that it, indeed, will be larger than average in size, it is inferred that *Emax* should follow in about 3 years, making the year 1999 as the expected *Emax*. If true, then *EM* would be expected to occur before April 2000, based upon the distribution of *EM* dates of occurrences relative to *Emax* occurrence (fig. 4), very probably sometime during 1999. However, presuming an *Em* of December 1996, it is inferred that *EM* will occur before December 2000 and, because large-amplitude cycles tend to be fast risers (the Waldmeier effect), one expects *EM* probably to occur several months prior to December 2000. Because the average *ASC* for a fast-rising cycle is about 41 months (±7 months), it is inferred that *EM* should occur after September 1999, probably near May 2000 (in agreement with Schatten et al.[15]). However, because the uncertainty in period length is ±3 months for short-period cycles, it is noted that *Em* for cycle 23 could come as early as September 1996 or as late as March 1997, inferring that *EM* could come as early as July 1999 to as late as October 2000. Resolution of this uncertainty will come once *Em* for cycle 23 has occurred and the early-rise portion of the cycle (Hathaway et al.[7]) has been experienced.

In closing, note that this study has pointed out the importance of sunspot minimum in the prediction of maximum amplitude (and its timing), and several methods have been identified that utilize either the occurrence or size of cycle minimum to forecast the size (maximum amplitude) of the ensuing cycle. In particular, these methods strongly suggest that cycle 23 onset is imminent, that cycle 22 is a short-period cycle, that maximum amplitude of cycle 23 will be larger than average, and that cycle 23 probably will be a fast-rising cycle. Because on-orbit construction of the *International Space Station* is scheduled to begin in late 1997 and will require several years to be completed, it is apparent that this activity will be concurrent with the volatile rising phase of cycle 23 through its maximum phase, which should persist through the year 2001. Thus, mission planners should pay very close attention to solar conditions as they unfold, especially between the years 1998 and 2001.

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Table 1. Summary of selected parameters for sunspot cycles 0 to 22.

SCN	Em	EM	Rm	RM	ASC	DES	PER	Emin	Emax	Rmin	Rmax	asc	des	per
0	–	Apr 1750	–	92.6	–	59	–	–	1750	–	83.4	–	5	–
1	Mar 1755	Jun 1761	8.4	86.5	75	60	135	1755	1761	9.6	85.9	6	5	11
2	Jun 1766	Sep 1769	11.2	115.8	39	69	108	1766	1769	11.4	106.1	3	6	9
3	Jun 1775	May 1778	7.2	158.5	35	76	111	1775	1778	7.0	154.4	3	6	9
4	Sep 1784	Feb 1788	9.5	141.2	41	123	164	1784	1787	10.2	132.0	3	11	14
5	May 1798	Feb 1805	3.2	49.2	81	66	147	1798	1804	4.1	47.5	6	6	12
6	Aug 1810	May 1816	0.0	48.7	69	84	153	1810	1816	0.0	45.8	6	7	13
7	May 1823	Nov 1829	0.1	71.7	78	48	126	1823	1830	1.8	70.9	7	3	10
8	Nov 1833	Mar 1837	7.3	146.9	40	76	116	1833	1837	8.5	138.3	4	6	10
9	Jul 1843	Feb 1848	10.5	131.6	55	94	149	1843	1848	10.7	124.7	5	8	13
10*	Dec 1855	Feb 1860	3.2	97.9	50	85	135	1856	1860	4.3	95.8	4	7	11
11	Mar 1867	Aug 1870	5.2	140.5	41	100	141	1867	1870	7.3	139.0	3	8	11
12	Dec 1878	Dec 1883	2.2	74.6	60	75	135	1878	1883	3.4	63.7	5	6	11
13	Mar 1890	Jan 1894	5.0	87.9	46	96	142	1889	1893	6.3	85.1	4	8	12
14	Jan 1902	Feb 1906	2.6	64.2	49	90	139	1901	1905	2.7	63.5	4	8	12
15	Aug 1913	Aug 1917	1.5	105.4	48	72	120	1913	1917	1.4	103.9	4	6	10
16	Aug 1923	Apr 1928	5.6	78.1	56	65	121	1923	1928	5.8	77.8	5	5	10
17	Sep 1933	Apr 1937	3.4	119.2	43	82	125	1933	1937	5.7	114.4	4	7	11
18	Feb 1944	May 1947	7.7	151.8	39	83	122	1944	1947	9.6	151.6	3	7	10
19	Apr 1954	Mar 1958	3.4	201.3	47	79	126	1954	1957	4.4	190.2	3	7	10
20	Oct 1964	Nov 1968	9.6	110.6	49	91	140	1964	1968	10.2	105.9	4	8	12
21	Jun 1976	Dec 1979	12.2	164.5	42	81	123	1976	1979	12.6	155.4	3	7	10
22	Sep 1986	Jul 1989	12.3	158.5	34	–	–	1986	1989	13.4	157.6	3	–	–
23								1996?						

* Beginning of modern sunspot era (i.e., complete daily sunspot records).

Legend:
  SCN is sunspot cycle number.
  Em is the epoch of sunspot minimum based on smoothed sunspot number (SSN).
  EM is the epoch of sunspot maximum based on SSN.
  Rm is the minimum amplitude based on SSN.
  RM is the maximum amplitude based on SSN.
  ASC is the ascent duration in months (minimum to maximum) based on SSN.
  DES is the descent duration in months (maximum to minimum) based on SSN.
  PER is the period in months (cycle minimum to minimum, equal to ASC + DES).
  Emin is the epoch of sunspot minimum based on the annual average (AA).
  Emax is the epoch of sunspot maximum based on the AA.
  Rmin is the minimum amplitude based on the AA.
  Rmax is the maximum amplitude based on the AA.
  asc is the ascent duration in years (minimum to maximum) based on the AA.
  des is the descent duration in years (maximum to minimum) based on the AA.
  per is the period in years (cycle minimum to minimum, equal to asc + des).
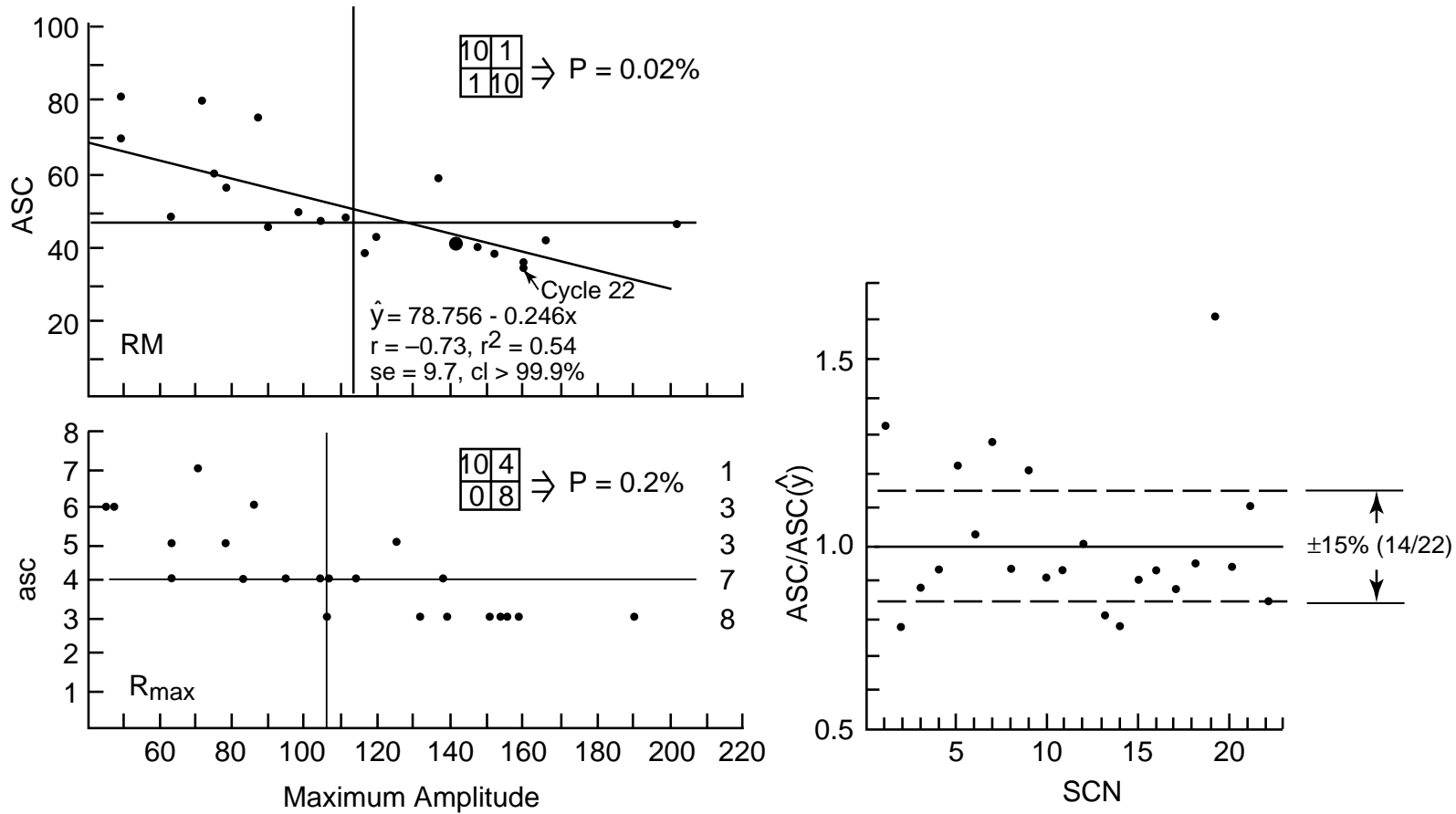
Figure 1. The Waldmeier effect. Plotted at the top-left is the ascent duration (*ASC*) in months versus maximum amplitude (*RM*) and at the bottom-left the ascent duration (*asc*) in years versus maximum amplitude (*Rmax*). The vertical and horizontal lines are the median values of the parameters. The diagonal line is the regression fit, with the correlation coefficient r, coefficient of determination r squared, standard error of estimate se, and confidence level cl identified. The results of Fisher's exact test for the 2 by 2 contingency tables are given. The frequency of occurrence of specific asc values is also shown. To the right is plotted the deviation of the ASC, as a ratio (observed ASC divided by the predicted ASC from the regression ŷ). Notice that the observed value usually is within 15% of the predicted value. SCN refers to the sunspot cycle number.

The Amplitude – Period Effect

All Cycles, Except 16, 19, and 21 (N=18)

$\hat{y} = 384.125 - 2.062x$
$r = -0.83$, $r^2 = 0.70$
$se = 21.0$, $cl > 99.9\%$

$\begin{array}{|c|c|}\hline 8 & 3 \\ \hline 2 & 8 \\ \hline \end{array} \Rightarrow P = 2.3\%$

Modern Cycles Only (N=13)

$\hat{y} = 345.719 - 1.711x$
$r = -0.40$, $r^2 = 0.16$
$se = 39.0$, $cl < 90\%$

All Cycles (N=21)

$\hat{y} = 370.446 - 1.930x$
$r = -0.66$, $r^2 = 0.44$
$se = 32.1$, $cl > 99.8\%$

$\hat{y}$ (Modern)
$\hat{y}$ (All)
$\hat{y}$ (All except 16, 19, 21)

Maximum Amplitude of Cycle n (RM)

Period of Cycle n-1 (months)

R_{max}

RM/$\hat{y}$(Modern)

±30% (9/13)

RM/$\hat{y}$(all, except 16, 19, 21)

±30% (16/21)

RM (Period of Cycle n-1)
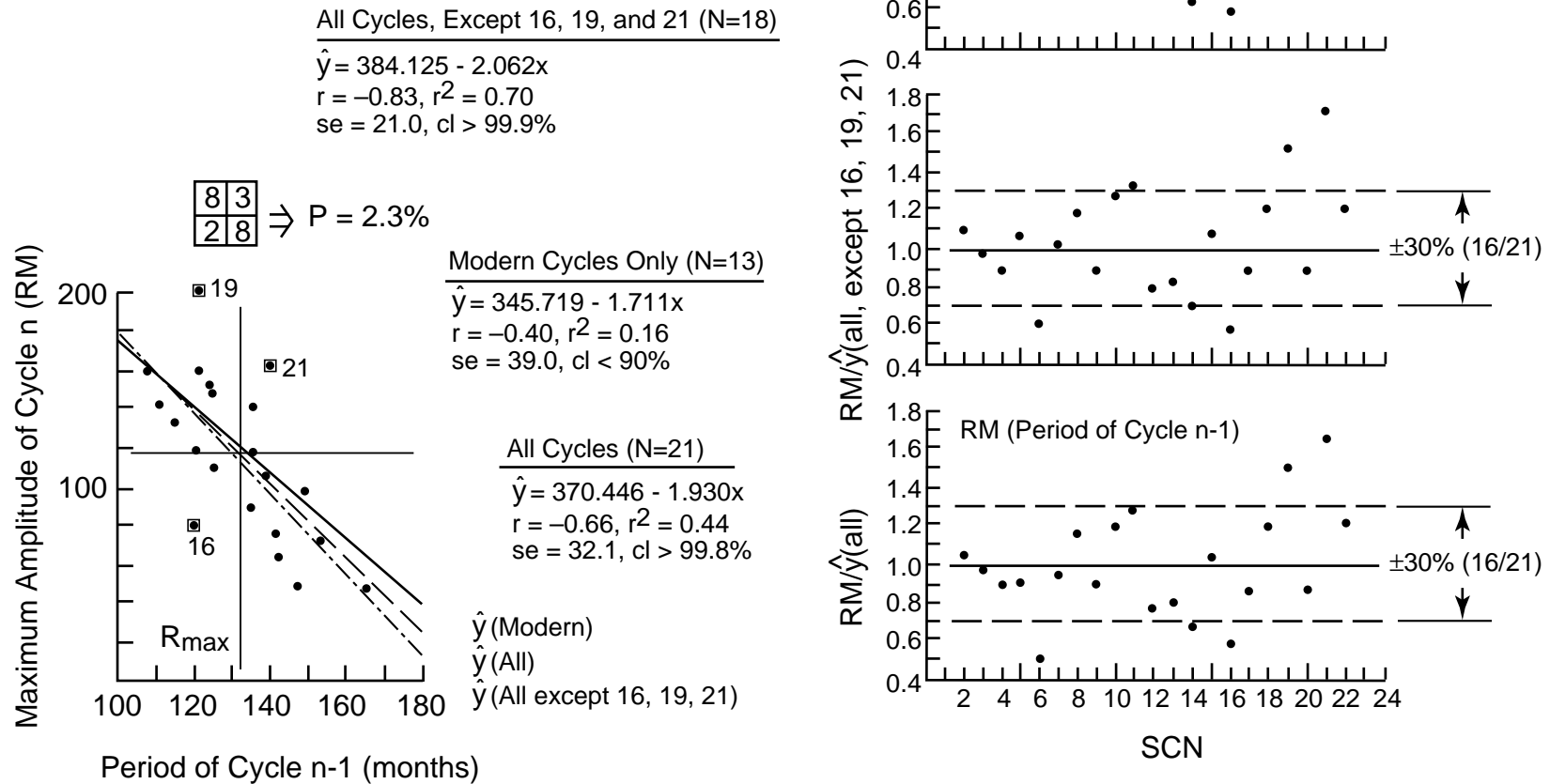
RM/$\hat{y}$(all)

±30% (16/21)

SCN

Figure 2. The amplitude-period effect. Three linear regressions are shown: all cycles, modern cycles only (i.e., cycles 10 to 22, and all cycles except cycles, 16, 19, and 21 (which clearly deviate the most from the inferred regressions). The chart follows a construction similar to that used in figure 1.

The Maximum – Minimum Effect

All Cycles, Except Cycle 19 (N=21)

$\hat{y} = 67.565 + 6.915x$
$r = 0.72$, $r^2 = 0.52$
$se = 26.4$, $cl > 99.9\%$

3	8
8	3

$\Rightarrow$ P = 4.3%

All Cycle (N=22)

$\hat{y} = 78.017 + 6.003x$
$r = 0.56$, $r^2 = 0.31$
$se = 35.1$, $cl > 99\%$

⊡ 19

$\hat{y}$ (Modern)

$\hat{y}$ (All except 19)

$\hat{y}$ (All)

Modern Cycles Only (N=13)

$\hat{y} = 89.771 + 5.243x$
$r = 0.47$, $r^2 = 0.22$
$se = 37.9$, $cl < 90\%$

Maximum Amplitude of Cycle n (RM)

Minimum Amplitude of Cycle n (Rm)

RM/$\hat{y}$(Modern)

±30% (10/13)

RM/$\hat{y}$(all, except 19)

±30% (15/22)

RM (Rm of Cycle n)

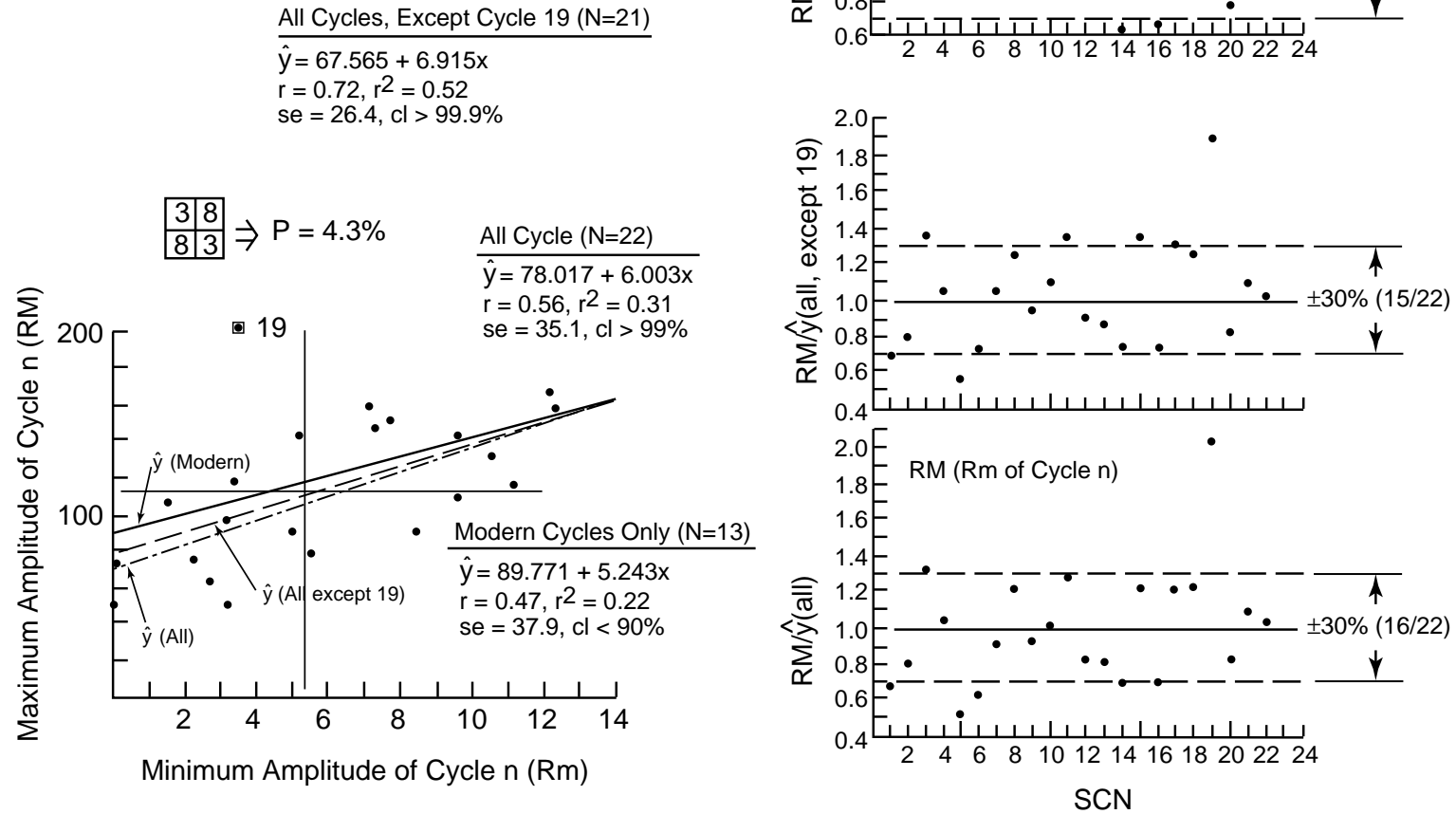RM/$\hat{y}$(all)

±30% (16/22)

SCN

Figure 3. The maximum-minimum effect. Three linear regressions are shown: all cycles, modern cycles only, and all cycles except cycle 19 (which clearly deviates the most from the regressions). The chart follows a construction similar to that used in figures 1 and 2.
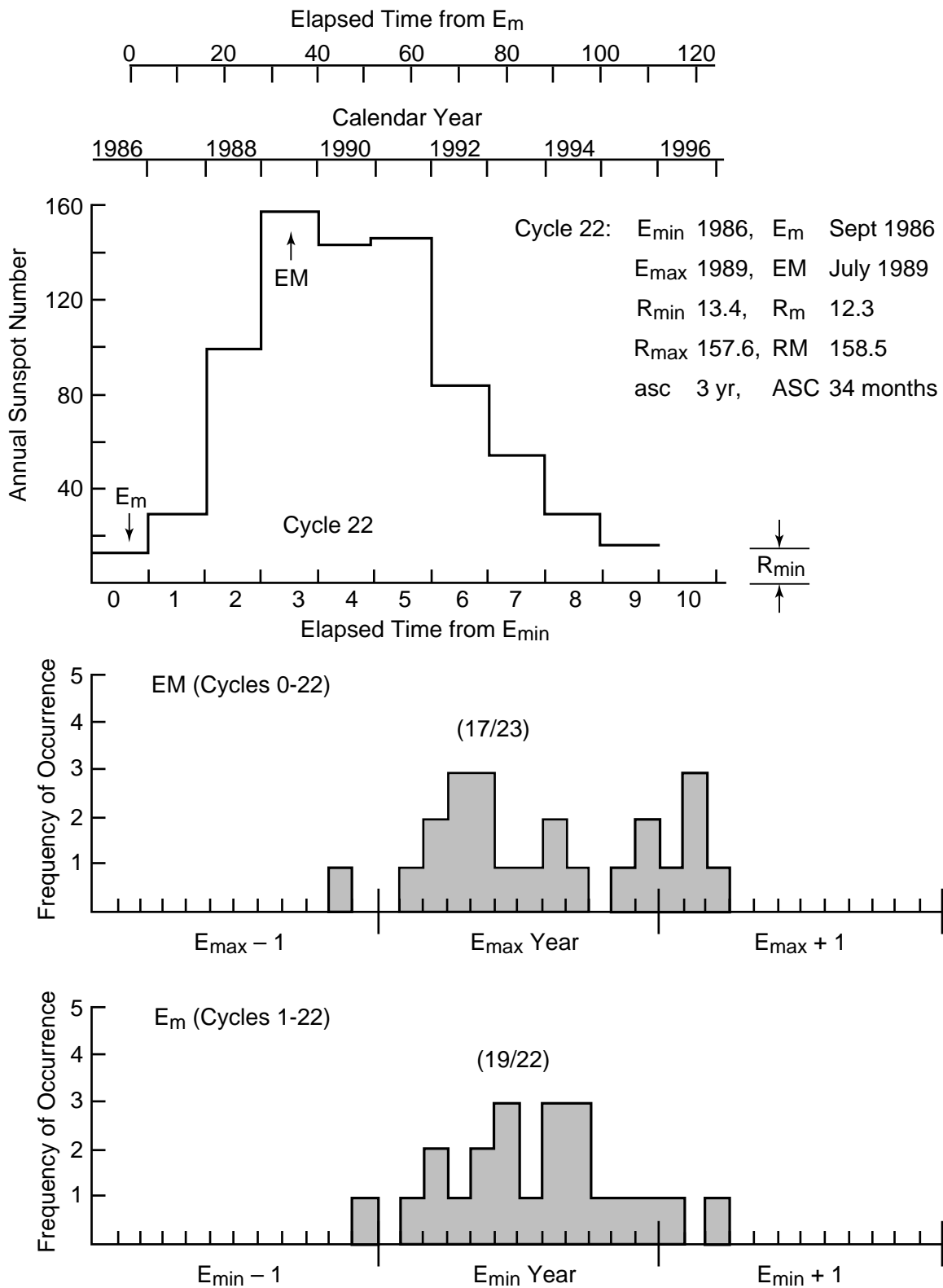
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Figure 4. Cycle 22—1986 to 1995, and the relative placements of smoothed sunspot number minimum and maximum near *Emin* and *Emax* years.
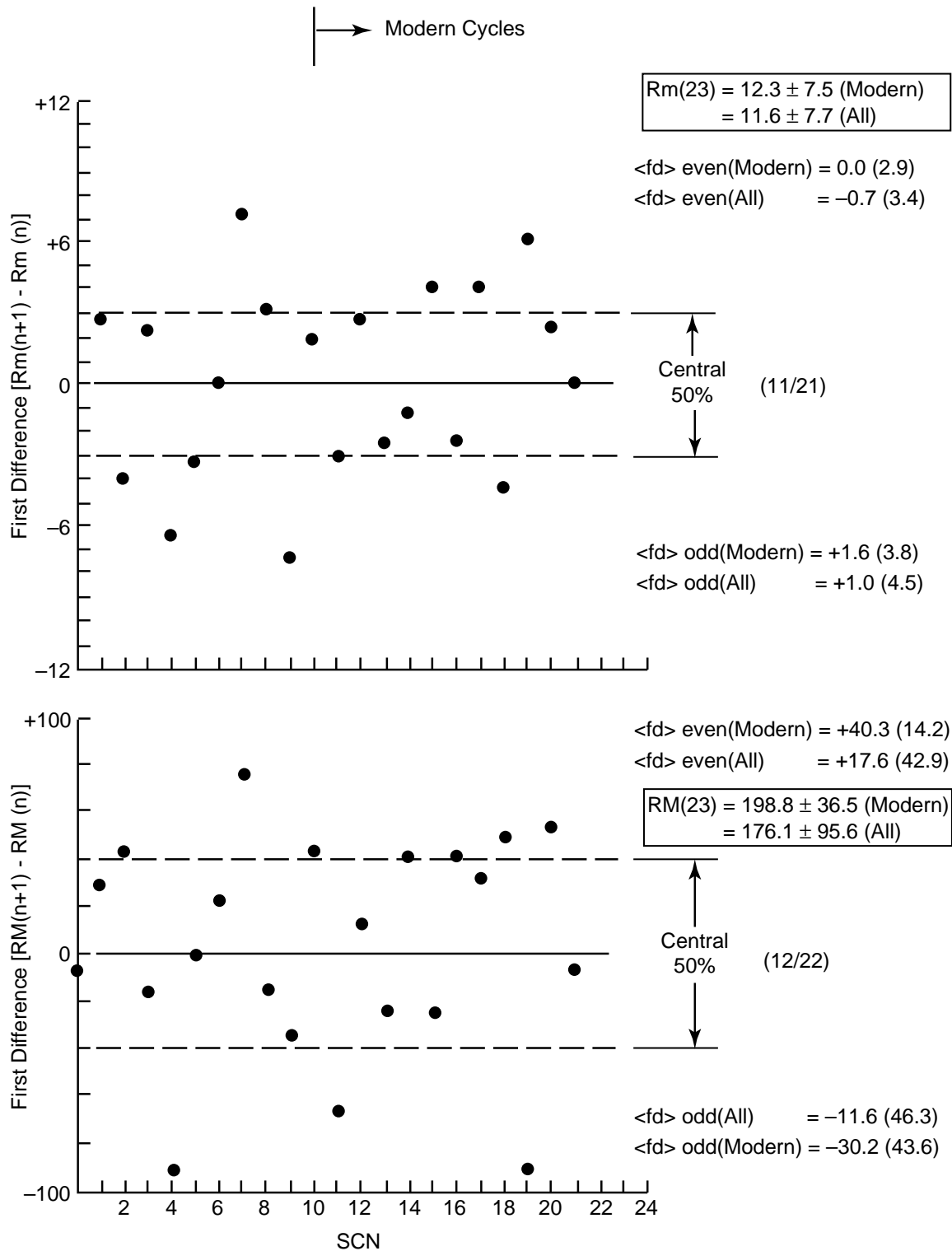
Figure 5. First differences (*fd*) of *Rm* and *RM* for cycles 0 to 21. Average *fd* is shown for *Rm* and *RM* based on even- and odd-numbered cycle grouping for all cycles and modern cycles only. The 95-percent level of confidence prediction intervals are shown for both *Rm* and *RM*, as applied to cycle 23.